ELIZADE UNIVERSITY, ILARA-MOKIN, ONDO STATE, NIGERIA

DEPARTMENT OF AUTOMOTIVE ENGINEERING

FIRST SEMESTER EXAMINATIONS

2020/2021 ACADEMIC SESSION

COURSE:	GNE 415 – Engineering Analysis (3 Units)
CLASS:	400 Level General Engineering
TIME ALLOWED:	2 Hours 30 Minutes
INSTRUCTIONS:	Attempt any **FIVE** questions **(125 marks)**

HOD'S SIGNATURE

Date: March, 2021

Question 1 (25 marks)

a) Four cards are drawn from an ordinary deck and not replaced. Find the probability of these event:
   i. That at least 1 ace is drawn
   ii. Getting 4 jacks
   iii. Getting an ace, a king, a queen and a 10
   iv. Getting a club, a spade, a heart and a diamond

   ...5 marks

b) Given that $f(z) = az^2 + i(bz) + c + i$. Find:
   i. $f'(z)$

   ...2.5 marks

   ii. $f'(i)$

   ...2.5 marks

c) Given that $f(z) = \frac{2z-i}{z+2i}$. Find:
   i. $f'(z)$

   ...2.5 marks

   ii. $f'(-1)$

   ...2.5 marks

d) Find the derivative of :
   i. $w = (z - 3i)^{(4z+2)}$

   ...10 marks

Question 2 (25 marks)

a) The table below shows Forest Fires and Acres Burned:

Fires (x)	72	69	58	47	84	62	57	45
Acres (y)	62	41	19	26	51	15	30	15

   i. Compute the values of correlation coefficient for the data

ii.   Find the equation of regression line for the data
iii. Plot a graph showing the given data and linear regression model in (b) above.

...20 marks

b) The American Automobile Association (AAA) reports that of the fatal car and truck accidents, 54 % are caused by car driver error. If 3 accidents are chosen at random, find the probability that
i.   All are caused by car driver error
ii.   None is caused by car driver error
iii. At least 1 is caused by car driver error.

...5 marks

Question 3 (25 marks)

a) Provide brief answers to the following questions:
i.   What is the difference between an outcome and an event?
ii.   Write the Laplace Transform of $f'(t)$ and $f''(t)$ for negligible initial conditions.

...5 marks

b) **Answer either sub question (i) or (ii):**
i.   Solve the given initial value problem using Laplace Transform:

$$\alpha_1 y'' + \alpha_2 y' + \alpha_3 y = te^{1/\infty}$$
$$y(0) = 1; y'(0) = -2$$

where,

$$\alpha_1 - \alpha_3 = \alpha_2 = 0$$
$$\alpha_1 = e^{1/\infty} = 1$$

...20 marks

ii.   Given that a function is defined as

$$f(t) = \begin{cases} 0 & \cdots\cdots\cdots & t < -\dfrac{1}{2} \\ 1 & \cdots\cdots\cdots & -\dfrac{1}{2} < t < \dfrac{1}{2} \\ 0 & \cdots\cdots\cdots & \dfrac{1}{2} < t \end{cases}$$

And Fourier Transform of $f(t)$ is defined as

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$$

Show that

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \operatorname{sinc}\left(\frac{\omega}{2}\right)$$

where,

$$\operatorname{sinc}\left(\frac{\omega}{2}\right) = \frac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}$$

...20 marks

Question 4 (25 marks)

a) Write an expression that can be used to test if function $f(z) = u(x,y) + iv(x,y)$ is harmonic function.

...5 marks

b) Show that $v = \alpha(e^{-x}x\sin y - e^{-x}y\cos y)$ is harmonic; where α is an arbitrary constant.

...10 marks

c) Prove the Cauchy's theorem.

...10 marks

Question 5 (25 marks)

a) The increase in cigarette taxes for 17 states in a six-month period are: 60, 20, 40, 40, 45, 12, 34, 51, 30, 70, 42, 31, 69, 32, 8, 18, 50. Find the standard variation.

...5 marks

b) Solve the given initial value problem using Laplace Transform:
$$\eta_1 y'' + \eta_2 y' + \eta_3 y = \eta_4 e^{\eta_5 t}$$
$$y(0) = y'(0) = 0$$

where,
$$\eta_1 = \eta_4 = -\eta_5 = 1$$
$$\eta_1 + \eta_4 = \eta_3$$
$$\eta_2 = 3\eta_1$$

...20 marks

Question 6 (25 marks)

a) Determine whether these events are mutually exclusive.
   i.   Roll a die: Get an even number, and get a number less than 3.
   ii. Roll a die: Get a prime number (2, 3, 5), and get an odd number.
   iii. Select a student in your class: The student has blond hair, and the student has blue eyes.
   iv. Select any course: It is a calculus course, and it is an English course
   Select a register voter: The voter is a Republican, and the voter is a Democrat.

...5 marks

b) Given that:
$$\alpha(z, \bar{z}) = A(x, y) + iB(x, y)$$

Show from first principles that:
$$2\frac{\partial \alpha}{\partial \bar{z}} = \left(\frac{\partial A}{\partial x} - \frac{\partial B}{\partial y}\right) + i\left(\frac{\partial A}{\partial y} + \frac{\partial B}{\partial x}\right)$$

...20 marks

Question 7 (25 marks)

a) Write the Cauchy-Riemann equations for the function:
$$f(z) = u(x, y) + iv(x, y)$$

...5 marks

b) Determine:
   i. $\int 2 \sin \beta z \cos \beta z \, dz$

...10 marks

   ii. $\int 3 \cot(\eta z + \lambda) \, dz$

...10 marks